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ABSTRACT

College students attempted to learn the concept of binomial probability from four-lesson teaching booklets. The booklets were of two forms, one emphasizing calculation with a formula and the other stressing the meanings of component variables. Also some subjects studied only one or two parts of the booklets while others completed the booklets before taking a transfer posttest. Results indicate a consistent treatment-by-posttest interaction pattern. The subjects using the calculating-with-a-formula approach excelled on near-transfer while the other group excelled on far-transfer. (Author/LS)

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ACQUISITION PROCESSES FOR MATHEMATICAL KNOWLEDGE

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Acquisition Processes for Mathematical Knowledge[1]

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In two main experiments Ss learned the concept of binomial probability in expository three-part booklets which either emphasized calculating with a formula (Sequence F) or the meanings of component variables (Sequence G). After studying either one, two or all three parts of their respective booklets, Ss were given a multileveled transfer posttest consisting of both near and far transfer items. The results indicated that at all three points in learning there was a clear and consistent pattern of treatment x posttest interaction (TPI) in which Sequence F Ss excelled on near transfer and Sequence G Ss excelled on far transfer, suggesting that there was no structural change in what is learned over the course of learning.

Introduction: Previous studies (Mayer & Greeno, 1972; Egan & Greeno, 1973) have suggested that teaching S to solve mathematical problems by different instructional methods may result in final learning outcomes which differ in structural or qualitative ways. This inference was based on a pattern of treatment x posttest interaction (TPI) in which subjects in one group excelled on one type of posttest item and subjects in another group excelled on different items. For example, Ss receiving an instructional method emphasizing algorithmic calculation or rule learning excelled on near transfer test items such as plugging values into the formula while Ss receiving instructional methods which emphasized conceptual understanding or discovery learning excelled on far transfer problems such as answering questions about the formula or solving story problems. A new question dealt with in the present study was: How can we characterize the acquisition processes which result in structurally different learning outcomes?

At least two kinds of theories of the acquisition process seem possible. (1) A fairly straight-forward idea, one which follows from asking "how much" is learned, is that apparent differences in what is learned are due to some Ss acquiring more of one kind of content and less of another relative to other Ss. (2) A more complex proposition, one that follows from asking "what" is learned (e.g., Roughead & Scandura, 1968), is that different kinds of learning outcomes are due to acquisition processes in which content material is encoded within different assimilative sets by different Ss. Although the first proposal requires only an analysis of the amount of material presented (and attended to), most recent theories of instruction have relied on modified versions of the second proposal in which S's cognitive activity or set during learning as well as the material presented determine the outcome of learning (e.g., see Ausubel, 1968). The present study was intended to provide information concerning the nature of acquisition with particular interest in distinguishing a process of "adding" new material to cognitive structure vs. a process of "integrating" new material within existing cognitive structure.

Method: The 108 Ss in Exp. I, 108 Ss in Exp. II, and 36 Ss in the Supplemental Study were University of Michigan students who volunteered to participate in psychological experiments for pay. In each experiment, S served in one cell of a 2 x 3 design, with the first factor being instructional sequence (Sequence F or Sequence G) and the second factor being amount of instruction (Amount I, Amount II or Amount III). All Ss received either a transfer posttest (in Exp. I and II) or instructions to reproduce what was taught (in the Supplemental Study), so comparisons between kinds of test items or kinds of protocol responses are within-subject comparisons.

In each study, the concept of binomial probability was taught in expository four-lesson teaching booklets, by two instructional methods which differed in sequencing and emphasis. One instructional method (Sequence F) began each lesson with a formal statement of the rule or subrule and explained component variables only within the context of calculating with the formula; the other method (Sequence G) began each lesson by attempting to relate component variables to Ss general experience, e.g., with "trials", "outcomes" and "successes", before presenting any formal statement of the rule. (See Table 1).

Learning was assessed, in Exp. I and Exp. II, by a 30-item multileveled transfer posttest which consisted of both near and far transfer items. Remoteness of transfer was varied in five problem types, three problem contents, and two problem formats, and each test item represented one cell in this 5 x 3 x 2 design. (See Table 2.)

To provide information on the acquisition question, the posttest was administered at three points in learning for Ss in both instructional groups. Some Ss were tested after reading all four lessons consisting of introduction, combinations, joint probability and binomial probability (Amount III), some after reading the first three lessons (Amount II) and some after reading just the first two lessons (Amount I). Both Exp. I and Exp. II used this procedure although the position of the combinations and joint probability lessons was reversed in Exp. II. In addition, a Supplemental Study was conducted in which S, instead of taking a transfer posttest after reading the appropriate number of lessons in his booklet, was asked to reproduce what he had just read as if he were explaining it to a naive learner.

Results: The main results with respect to the acquisition process were as follows: (1) The overall proportion correct on the posttest increased significantly for both instructional groups as the amount of instruction, i.e., the number of lessons presented, increased (Exp. I: effect of amount instruction, $F = 4.49$, $df = 2/29$, $p < .025$; Exp. II: effect of amount of instruction, $F = 22.10$, $df = 2/96$, $p < .001$). See Figures 1 and 2.
(2) Treatment x Posttest interaction was generally present for all three posttest dimensions in both experiments, with Sequence F excelling on Familiar and Transformed Types, Binomial Content, and Formula Format problems and Sequence G excelling on Unanswerable and Question Types, Joint Probability and Binomial Contents, and Story Format problems (Exp. I: Sequence x format

interaction, $F = 14.72$, $df = 1/96$, $p < .001$; sequence x type, $F = 6.28$, $df = 4/384$, $p < .001$; sequence x content, $F = 10.19$, $df = 2/192$, $p < .001$; Exp. II: sequence x format, $F = 21.65$, $df = 1/96$, $p < .001$; sequence x type, $F = 1.63$, $df = 4/384$, $p > .15$; sequence x content, $F = 13.09$, $df = 2/92$ $p < .001$). These findings add replicative support to previous findings (Mayer & Greeno, 1972). See Figures 1 and 2. (3) The pattern of TPI did not reliably change from Amount I to Amount II to Amount III for format or type of posttest item in either experiment, and there was no consistent Treatment x Posttest x Amount interaction required to reject the hypothesis that the same structural differences were present at all three points in learning (Exp. I: Sequence x format x amount, $F = 1.00$, $df = 2/96$, p = ns; sequence x type x amount, $F = 1.03$, $df = 8/384$, p = ns; sequence x content x amount, $F = 1.77$, $df = 4/192$, p = ns; Exp. II: sequence x format x amount, $F = 2.49$, $df = 2/96$, p = ns; sequence x type x amount, $F = 1.17$, $df = 3/384$, p = ns; sequence x content x amount, $F = 2.62$, $df = 4/192$, $p < .05$). See Figures 1 and 2. (4) The combinations lesson was far more important in producing increases in posttest performance than the joint probability lesson, perhaps because it was less familiar to the Ss. However, this was expecially true for the Sequence F Ss in Experiment II (Sequence x amount interaction, $F = 3.30$, $df = 2/96$, $p < .05$), suggesting that Ss receiving Sequence G were better able to create solutions after being exposed to only part of the material than were Ss in Sequence F. See Figure 2. (5) In the supplemental study, as the amount of instruction (i.e., the number of lessons) increased, the number of words and symbols output in S's protocol increased for Ss in Sequence F but remained about the same or decreased for Ss in Sequence G (For words, effect of instructional sequence $F = 12.97$, $df = 1/30$, $p < .005$; sequence x amount interaction, $F = 5.10$, $df = 2/30$, $p < .025$; for symbols, effect of instructional sequence, $F = 9.25$, $df = 1/30$, $p < .001$; sequence x amount interaction, $F = 5.63$, $df = 2/30$, $p < .01$; for both: effect of sequence $F < 1.00$, $df = 1/30$, $p = ns$; sequence x amount interaction, $F = 6.91$, $df = 2/30$, $p < .005$). See table 3.

Discussion: These results again demonstrate that there may be qualitative, as well as quantitative, differences in what is learned by Ss from mathematics text. Behavioral objectives, or other quantitative measures, which do not take into account the breadth and quality of S's transfer ability, may actually encourage a kind of learning outcome (exhibited by Ss in Sequence F) which is less likely to support further meaningful learning.

The results also provide information concerning the acquisition processes which result in structurally different learning outcomes. If subjects in different treatments were simply adding different material to memory during learning, the expected outcome would be a steady increase in the strength of TPI as amount of instruction progressed, a steady increase in the length of reproduction protocols for both treatments as the amount of instruction progressed, and no difference between the groups in their ability to create novel solutions based on only a part of the material.

However, a second idea is that subjects in different instructional treatments were, instead, evoking different assimilative sets or encoding techniques quite early in learning and integrating new material within these sets throughout learning. For example, Sequence G Ss were integrating material within a rich bank of existing knowledge -- what Greeno (1972) calls "propositional knowledge" -- while Ss in Sequence F relied on a narrower assimilative set concerned with arithmetic computations and applying formulas ("algorithmic knowledge") and thus were more likely to add content material as presented to memory. This interpretation is most consistent with the observed results:

no structural change (i.e., no change in TPI) across learning since the different encoding techniquos were present throughout learning, a lengthening of reproduction protocols as amount of instruction progressed for Sequence F Ss who were "adding" material to memory but no such increase for Sequence G Ss who were integrating and streamlining new information, and Sequence G Ss better at creating solutions with only part of the instruction presented because they could use aspects of the rich assimilative set they had evoked.

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FOOTNOTES

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TABLE 1

SUMMARY OF TWO INSTRUCTIONAL BOOKLETS

	Sequence F	Sequence G
Introduction	Present binomial formula. Break it down into 3 smaller algorithms.	Discuss main variables (e.g., number of trials, successes, etc.) in relation to general experience.
Combinations	Present formula for combinations. Break down into smaller steps. Give example and solve by algorithm.	Re-discuss relevant concepts. Give examples solved by concepts. Present formula for combination.
Joint Probability	Present formula for joint probability. Break down into smaller steps. Give example solved by algorithm.	Re-discuss relevant concepts. Give example solved by concepts. Present formula for joint probability
Binomial Probability	Present complete formula, as product of formula in lesson 2 x lesson 3. Give example solved by algorithmic steps.	Re-discuss concepts which tie sub-formulas together. Give example. State binomial formula.

The formula for both groups was presented as:

$$P(R,N) = \frac{N!}{(N-R)!R!} \times P^R \times (1-P)^{N-R}$$

where P(R,N) is binomial probability, N is number of trials, R is number of successes, P is probability of success.

TABLE 2

EXAMPLES OF POSTTEST ITEMS

There were five problem types:

- F or Familiar: just like the examples presented in the text
- T or Transformed: require only a small change to be just like the examples in the text
- L or Luchins: look hard but are really easy if you think about it
- Q or Question: ask a question about the formula rather than computing a value
- U or Unanswerable: pose impossible or incomplete information

There were three content areas:

- B or Binomial: concern the entire formula
- C or Combinations: concern only part of the formula, i.e., the combinations sub-formula
- J or Joint: concern only a part of the formula, i.e., the joint probability sub-formula

There were two problem formats:

- F or Formula: stated in terms of N, R and P
- S or Story: stated in terms of a situation

Familiar Type, Formula Format, Binomial Content: $N = 4$, $R = 3$, $P = 20$. What is $P(R,N)$? The correct answer requires plugging the values of N, R and P into the formula to get,

$$P(R,N) = C(4,3) \times (.20)^3 \times (.80)^1 = 16/625 .$$

Transformed Type, Formula Format, Joint Content: $P = 3(1-P)$, $N = 6$, $R = N-R$. What is $P^R \times (1-P)^{N-R}$? The correct answer requires solving for P and R before plugging into the joint probability formula to get,

$$P^R \times (1-P)^{N-R} = (3/4)^3 \times (1/4)^3 = 27/4096 .$$

Luchins Type, Story Format, Combinations Content: There are 10 different sequences that have exactly two successes. All the sequences have the same length. How long are they? The correct answer requires finding a value of N to fit $C(N,R) = 10$ and $R = 2$, as shown:

$$\frac{N!}{(N-2)!2!} = 10 \quad N = 5$$

Question Type, Story Format, Binomial Content: Is there a difference between the probability that two dice rolled at once both come up 6 and the probability that one die rolled twice comes up 6 both times? The answer requires an understanding of independence of events; hence, the subject should answer "no" or "no difference."

Unanswerable Type, Story Format, Combinations Content: How many different sequences have the same number of successes as failures? The answer requires the recognition of insufficient information, i.e., no value of N is given, and hence the correct answer is "no answer".

TABLE 3

AVERAGE NUMBER OF ELEMENTS IN REPRODUCTION FOR TWO INSTRUCTIONAL GROUPS AND THREE AMOUNTS OF INSTRUCTION--SUPPLEMENTAL STUDY

Average Number of Words

Instructional Sequence	Amount of Instruction I	II	III	Average
F	110	152	243	168
G	259	278	206	247
Ave.	184	215	225	

Average Number of Symbols

Instructional Sequence	Amount of Instruction I	II	III	Average
F	95	187	190	157
G	111	107	88	102
Ave.	103	147	139	

Average Number of Words & Symbols

Instructional Sequence	Amount of Instruction I	II	III	Average
F	205	339	433	326
G	370	385	293	350
Ave.	287	362	363	

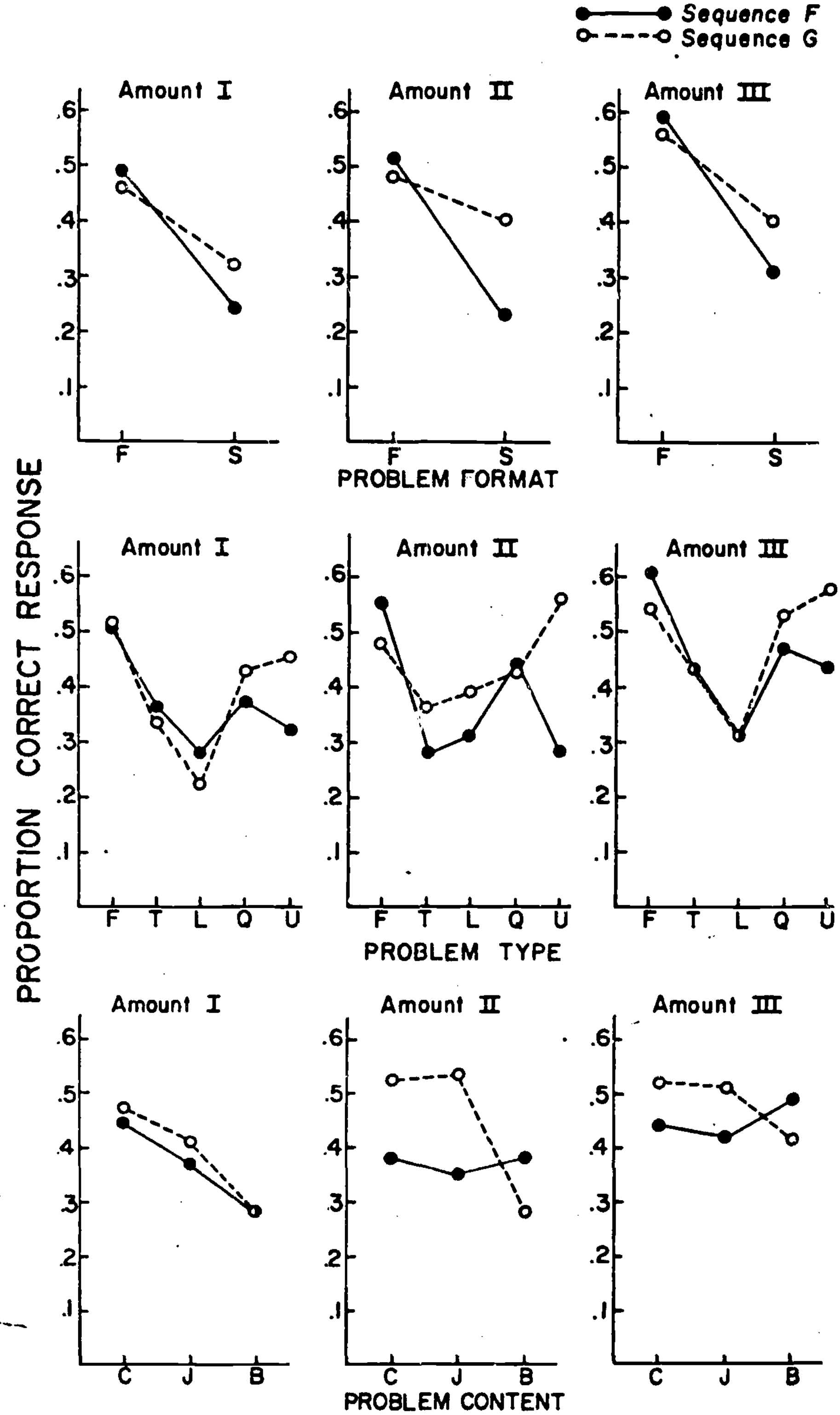

Fig. 1 Treatment x Posttest Interaction by Amount of Instruction for Three Posttest Dimensions -- Experiment I

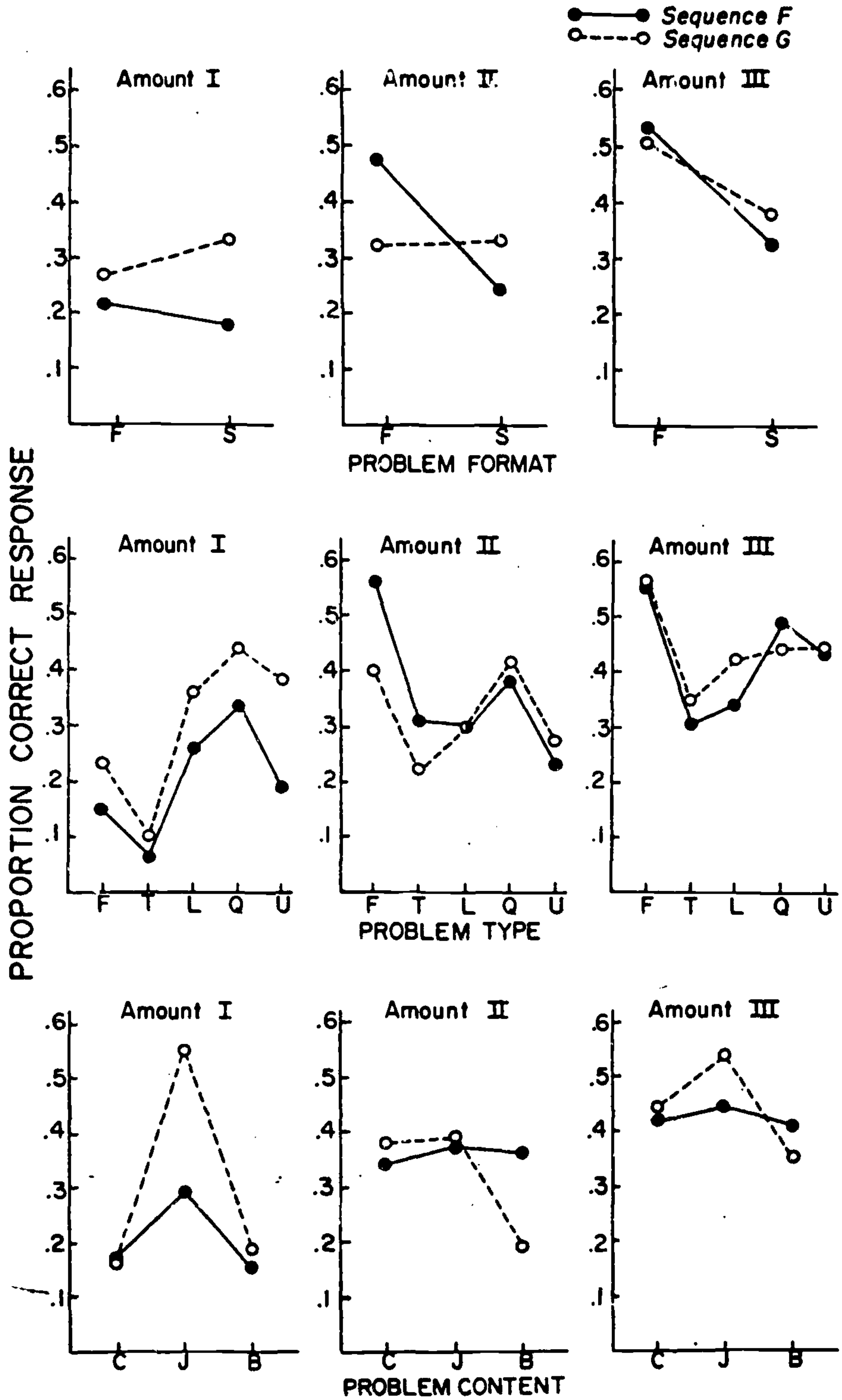

Fig. 2 Treatment x Posttest Interaction by Amount of Instruction for Three Posttest Dimensions -- Experiment II